



(i) $F = mg (\sin \theta + \mu \cos \theta) \cos \alpha$ (α = angle of friction)

(ii) $\delta = \alpha$, where $\alpha = \tan^{-1} \mu$

Connector 18: Find the force required to be applied parallel to the inclined plane, to push a 10 kg body up an incline of 30° with a constant acceleration of 5 m s^{-2} . The coefficient of friction is 0.2 (Take $g = 10 \text{ m s}^{-2}$)

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Laws of Motion 4.51

Solution:

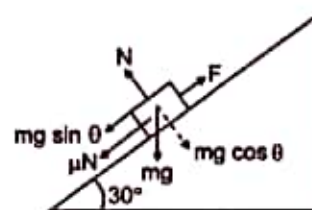
$$N = mg \cos \theta$$

$$F - (mg \sin \theta + \mu N) = ma$$

$$F = ma + mg \sin \theta + \mu N$$

$$= 10 \left(5 + 10 \times \frac{1}{2} + 0.2 \times 10 \times \frac{\sqrt{3}}{2} \right)$$

$$= 10 (5 + 5 + \sqrt{3}) = 117.3 \text{ N}$$



Connector 19: A child sits on a horse hung by a light rod of length r in a carousel of radius R . If the angle made by the horse with the vertical is 37° , when the carousel is rotating with a uniform angular speed, find the angular speed of the carousel.

Solution:

Consider the frame rotating with angular velocity ω

$$r' = R + r \sin 37^\circ$$

If 'T' is the tension in the rod,

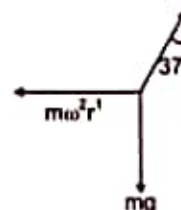
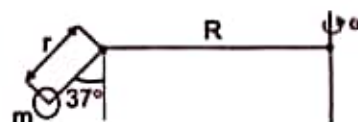
$$T \cos 37^\circ = mg \text{ and}$$

$$T \sin 37^\circ = m\omega^2 r' \Rightarrow \tan 37^\circ = \frac{m\omega^2 r'}{mg}$$

$$\frac{3}{4} = \omega^2 \frac{(R + r \frac{3}{5})}{g}$$

$$\omega^2 = \frac{3}{4} g \frac{5}{(5R + 3r)}$$

$$\omega = \sqrt{\frac{15}{4} \frac{g}{(5R + 3r)}}$$



Connector 20: A particle of mass 'm' travels in a vertical circular path of radius r under the action of a tangential force. If the speed of the particle at a position when the radius vector makes an angle ' θ ' with vertical as shown in figure, is $v = v_0 \sin \theta$, find the maximum normal reaction (given $v_0^2 = g r$).

Solution:

$$\frac{mv^2}{r} = N - mg \cos \theta$$

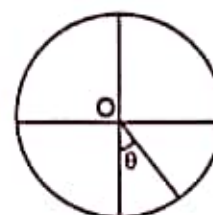
$$N = \frac{mv^2}{r} + mg \cos \theta = \frac{mv_0^2}{r} \sin^2 \theta + mg \cos \theta$$

$$\frac{v_0^2}{r} = g$$

$$\Rightarrow N = mg (\sin^2 \theta + \cos \theta) = mg (1 - \cos^2 \theta + \cos \theta)$$

$$= mg \left[\frac{5}{4} - \left(\cos \theta - \frac{1}{2} \right)^2 \right]$$

$$\Rightarrow N_{\max} = \frac{5}{4} mg$$



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4.50 Laws of Motion

$$\therefore F = \frac{\mu mg}{\frac{1}{\sqrt{1+\mu^2}} + \frac{\mu^2}{\sqrt{1+\mu^2}}} = \frac{\mu mg}{\sqrt{1+\mu^2}} = mg \sin \alpha$$

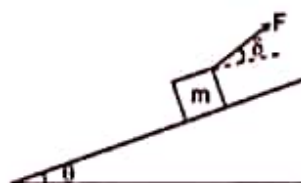
Answer

$$(i) F = \frac{\mu mg}{\sqrt{1+\mu^2}} = mg \sin \alpha \quad (\alpha = \text{Angle of friction})$$

$$(ii) \theta = \alpha = \tan^{-1} \mu$$

Connector 17: A body of mass m is kept on a rough inclined plane of angle θ and coefficient of friction μ .

- What is the minimum force F required to pull the mass up the inclined plane?
- What is the angle δ with respect to the inclined plane at which the force has to be applied?



Solution:

$$\text{Normal reaction } N = mg \cos \theta - F \sin \delta$$

$$\text{Max. frictional force } f = \mu N$$

In the limiting case, when F just pulls up the mass m ,

$$F \cos \delta = mg \sin \theta + f = mg \sin \theta + \mu mg \cos \theta - \mu F \sin \delta$$

$$\therefore F(\cos \delta + \mu \sin \delta) = mg \sin \theta + \mu mg \cos \theta$$

$$\therefore F = \frac{mg \sin \theta + \mu mg \cos \theta}{\cos \delta + \mu \sin \delta}$$

F is minimum when denominator is maximum

$$\therefore \frac{d}{d\delta}(\cos \delta + \mu \sin \delta) = 0 = -\sin \delta + \mu \cos \delta$$

$$\therefore \mu = \tan \delta = \tan \alpha \text{ where } \alpha \text{ is the angle of friction.}$$

$$\therefore \cos \delta = \frac{1}{\sqrt{1+\mu^2}}; \sin \delta = \frac{\mu}{\sqrt{1+\mu^2}}$$

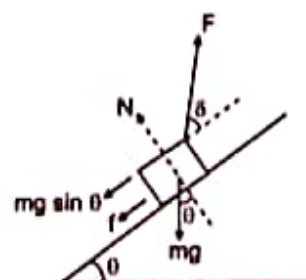
$$\text{and } F = \frac{mg \sin \theta + \mu mg \cos \theta}{\sqrt{1+\mu^2}}$$

$$\therefore F = (mg \sin \theta + \mu mg \cos \theta) \cos \alpha$$

Answer

$$(i) F = mg (\sin \theta + \mu \cos \theta) \cos \alpha \quad (\alpha = \text{angle of friction})$$

$$(ii) \delta = \alpha, \text{ where } \alpha = \tan^{-1} \mu$$

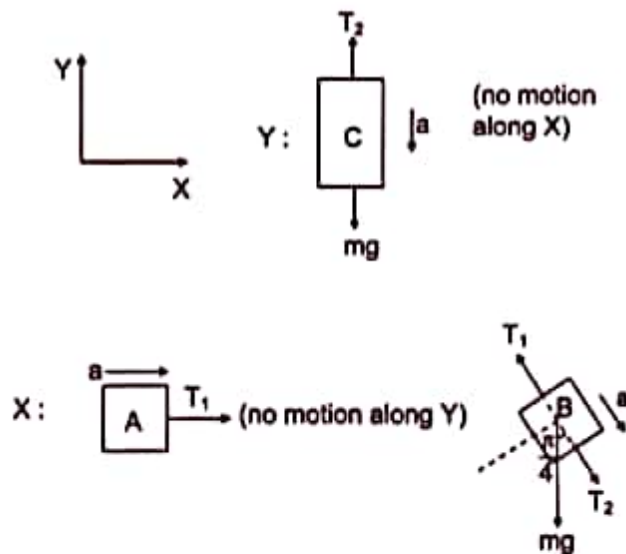


Connector 18: Find the force required to be applied parallel to the inclined plane, to push a 10 kg body up an incline of 30° with a constant acceleration of 5 m s^{-2} . The coefficient of friction is 0.2 (Take $g = 10 \text{ m s}^{-2}$)



4.46 Laws of Motion

Solution: Resolving forces along the tangent as strings are inextensible, a is the same.



For mass A,

$$T_1 = m a$$

For mass B,

$$T_2 - T_1 + mg \frac{1}{\sqrt{2}} = m a$$

For mass C,

$$mg - T_2 = m a$$

adding

$$3 m a = m g \left(1 + \frac{1}{\sqrt{2}} \right) \Rightarrow a = \frac{g \left(1 + \frac{1}{\sqrt{2}} \right)}{3}$$

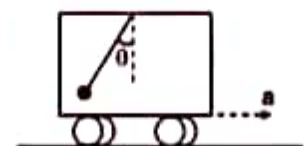
$$T_1 = m a = \frac{m g}{3} \left(1 + \frac{1}{\sqrt{2}} \right);$$

$$T_2 = m g - m a$$

$$= m g \left(1 - \frac{1}{3} \left(1 + \frac{1}{\sqrt{2}} \right) \right) = \frac{m g}{3} \left(2 - \frac{1}{\sqrt{2}} \right)$$

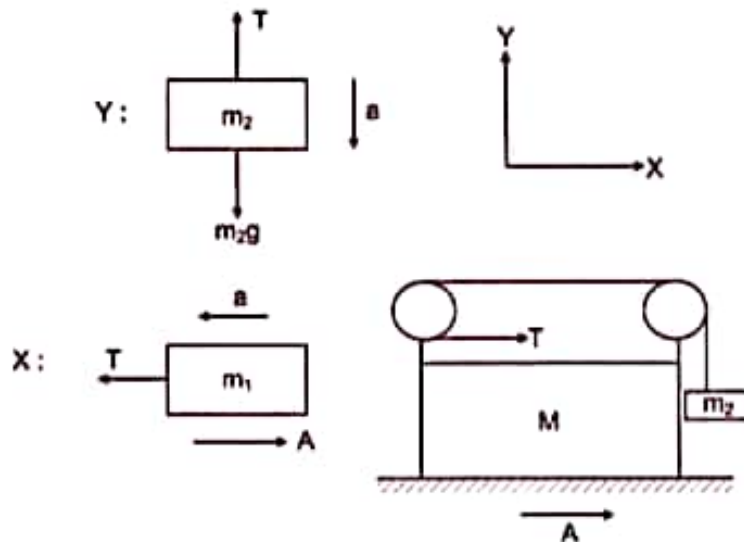
Connector 11: (i)

A body of mass m is suspended by a light inextensible thread from the ceiling of a truck, which moves with a horizontal acceleration ' a '. Find the angle θ , which the thread makes with the vertical at equilibrium.





Solution: F.B.D



$m_2g - T = m_2a$ (where a is the downward acceleration of m_2)

$$T = m_1(a - A)$$

$$(m_2 + M)A = T$$

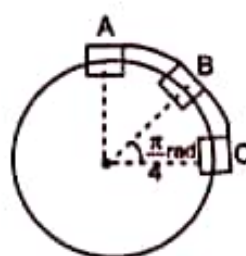
$$\Rightarrow \frac{T}{m_1} = a - A \text{ and } \frac{T}{m_2 + M} = A$$

$$\Rightarrow T \left(\frac{1}{m_1} + \frac{1}{m_2 + M} \right) = a$$

$$m_2g = m_2a + T = \left(m_2 + \frac{1}{\frac{1}{m_1} + \frac{1}{m_2 + M}} \right) a \Rightarrow$$

$$a = \frac{g}{1 + \frac{1}{\frac{m_2}{m_1} + \frac{m_2}{m_2 + M}}} = \left[\frac{m_1(m_2 + M)}{m_2(m_1 + m_2 + M)} + 1 \right]$$

Connector 10: Three equal masses are placed on a smooth vertical circular ring as shown and connected by strings. Find the initial acceleration of the masses and the tension in the strings, if the strings lie along the circumference.





Newton's law

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4.44 Laws of Motion

Solution: Let T be the tension in the string between m_1 and m_2 and T_1 the tension in the string between the bodies m_2 and m_3 . Also, let a be the acceleration. Then

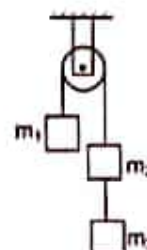
$$T - m_1 g = m_1 a \quad (1)$$

$$(m_2 + m_3)g - T = (m_2 + m_3) a \quad (2)$$

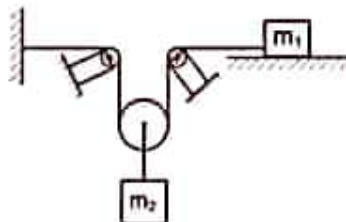
$$a = \left(\frac{m_2 + m_3 - m_1}{m_1 + m_2 + m_3} \right) g$$

$$m_1 g - T_1 = m_1 a \Rightarrow T_1 = m_1 (g - a) = \frac{m_1 g ((m_1 + m_2 + m_3 + m_1) - (m_2 + m_3))}{m_1 + m_2 + m_3}$$

$$T_1 = \frac{2m_1 m_1}{m_1 + m_2 + m_3} g$$



Connector 8: The pulleys are light, frictionless and the strings are light and inextensible. Find the accelerations of m_1 and m_2 . No friction anywhere.



Solution: F.B.D

Let acceleration of m_1 be a_1 and acceleration of m_2 be a_2 .

$$a_1 = 2a_2 \quad (\text{By constraint method})$$

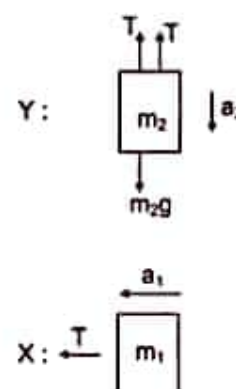
$$\text{For } m_2: m_2 g - 2T = m_2 a_2$$

$$\text{For } m_1: T = m_1 a_1$$

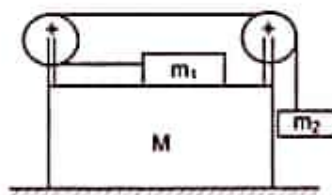
$$m_2 g = 2m_1 a_1 + m_2 a_2 = 2m_1 \times 2a_2 + m_2 a_2 = (4m_1 + m_2) a_2 \quad (\because a_1 = 2a_2)$$

$$a_2 = \left(\frac{m_2 g}{4m_1 + m_2} \right)$$

$$a_1 = \left(\frac{2m_2}{4m_1 + m_2} g \right)$$



Connector 9: Find the vertical acceleration of m_2 . Pulleys are massless and strings are light and inextensible. M moves to the right with acceleration A . There is no friction anywhere.



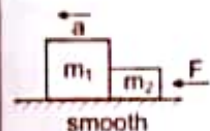
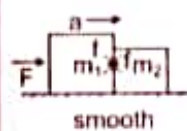
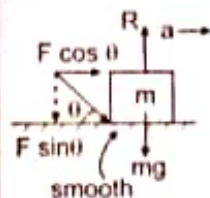
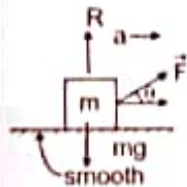
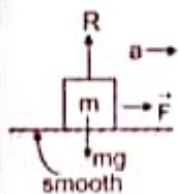


4.38 Laws of Motion

$$a = \frac{v^2}{r}$$

$$F = \frac{mv^2}{r}$$

$$\theta = \tan^{-1} \left(\frac{v^2}{rg} \right)$$



$a \rightarrow$ centripetal acceleration

$v \rightarrow$ linear velocity

$r \rightarrow$ radius of circular path

$F \rightarrow$ centripetal force

$m \rightarrow$ mass of particle

$\theta \rightarrow$ Banking angle or inclination of cyclist to the vertical

$v \rightarrow$ linear velocity

$r \rightarrow$ radius of curvature

$g \rightarrow$ acceleration due to gravity

$$R = mg$$

$R \rightarrow$ Normal reaction

$$a = \frac{F}{m}$$

$F =$ horizontal pull

$F =$ inclined pull

$$R = mg - F \sin \theta$$

$$a = \frac{F \cos \theta}{m}$$

$$R < mg$$

$F =$ inclined push

$$R = mg + F \sin \theta$$

$$a = \frac{F \cos \theta}{m}$$

$$R > mg$$

$$f = \frac{m_1 F}{m_1 + m_2}$$

$f \rightarrow$ force of contact

$$a = \frac{F}{(m_1 + m_2)}$$

$$f = \frac{m_1 F}{m_1 + m_2}, f \rightarrow \text{contact force}$$

$$a = \frac{F}{(m_1 + m_2)}$$

$$a = \frac{F}{m_1 + m_2}$$

$$T = \frac{m_1 F}{m_1 + m_2}$$



SUMMARY

$$p = mv$$

$$F = \frac{dp}{dt}$$

$$F = ma$$

$$F = v \frac{dm}{dt}$$

$$J = F\Delta t$$

$$R_1 = m(g+a)$$

$$R_2 = m(g-a)$$

$$F_s = \mu_s N$$

$$F_k = \mu_k N$$

$$\tan \theta = \mu_s$$

$$v = \frac{-m}{M} V$$

$$T = -u \frac{dm}{dt}$$

$$a_1 = \frac{1}{M} \frac{u dm}{dt}$$

$$a_2 = \frac{1}{M} \frac{u dm}{dt} - g$$

$p \rightarrow$ linear momentum, $m \rightarrow$ mass, $v \rightarrow$ velocity

$F \rightarrow$ applied force

$p \rightarrow$ momentum

$F \rightarrow$ applied force

$m \rightarrow$ mass (constant)

$a \rightarrow$ acceleration

$v \rightarrow$ velocity

$\frac{dm}{dt} \rightarrow$ rate of change of mass

$J \rightarrow$ impulse

$F \rightarrow$ force

$\Delta t \rightarrow$ time interval

$R_1 \rightarrow$ apparent weight when a body is accelerating upwards.

$a \rightarrow$ upward acceleration of the body.

$R_2 \rightarrow$ apparent weight when a body is accelerating downwards.

$a \rightarrow$ downward acceleration of the body.

$F_s \rightarrow$ Limiting friction.

$\mu_s \rightarrow$ coefficient of static friction.

$N \rightarrow$ normal reaction

$F_k \rightarrow$ Kinetic friction.

$\mu_k \rightarrow$ coefficient of kinetic friction.

$\theta \rightarrow$ angle of friction (angle of repose)

$\mu_s \rightarrow$ coefficient of static friction

$v \rightarrow$ recoil velocity of gun

$V \rightarrow$ velocity of bullet

$m \rightarrow$ mass of bullet

$M \rightarrow$ mass of gun

$T \rightarrow$ thrust on rocket

$u \rightarrow$ Relative speed of exhaust gas w.r.t rocket

$\frac{dm}{dt} \rightarrow$ rate of decrease of mass (or rate of fuel consumption)

$a_1 \rightarrow$ acceleration of rocket in the absence of gravitational force

$a_2 \rightarrow$ acceleration of rocket in the presence of gravitational force

$M \rightarrow$ mass of the rocket plus mass of fuel remaining